

Mathematical Preliminaries

Last Time

- Overview
- Introduction
- History
- Assumed background

Sets

- **A set:** a non-repeating, unordered collection of objects (elements, members)
 $A = \{C, Fortran, Lisp, C++, Java\}$
- **Empty set** \emptyset
- **Subset:** $A \subseteq B$, every element of A is an element of B
- **Cardinality** of a set: the number of elements in it
- **Set operations**
 - union, intersection, complement, difference
- **Power set** of a set A : 2^A , the set of all subsets of A
 $2^{\{2,3,5\}} = \{\emptyset, \dots\}$
- **Cartesian (cross) product** of two sets A and B : $A \times B$,
 $\{(a, b) | a \in A, b \in B\}$

Sequences

- *A sequence*: a list of objects in some order
- *Tuples*: a sequence of k elements is a k -tuple.
$$(a, x, 2)$$
- Repetition is allowed.

Functions

- A function maps an input to an output.
- Domain D : the set of possible inputs
- Range R : the set of outputs
- k -ary function: a function with k arguments.

$$f : D \longrightarrow R$$

Input: (a_1, a_2, \dots, a_k) , a k -tuple

- unary function if $k = 1$
- binary function if $k = 2$

Relations

- *Predicate (property)* : a function whose range is $\{\text{True}, \text{False}\}$.
- *Relation*: a property whose domain is A^k
k-ary relation, k-ary relation on A: $R(a_1, \dots, a_k)$
 - binary relation: aRb
- *equivalence relation*: two objects being equal
 - reflexive: $\forall x, xRx$
 - symmetric: $\forall x y, xRy$ iff yRx
 - transitive: $\forall x y z, xRy$ and $yRz \implies xRz$

Graphs

- A graph consists of a finite set of vertices with lines connecting some of them (edges).

$$G = (V, E)$$

- Undirected graph
 - *degree* of a node: the number of edges
 - *path* : a sequence of nodes connected by edges
 - *cycle*: a path starts and ends in the same node
 - *tree*: no cycle
- Directed graph:
 - *in-degree* and *out-degree*
 - *directed path*
 - *directed acyclic graph (DAG)*

Strings

- Strings of characters
- Alphabet : any finite set, Σ and Γ
 $\Sigma = \{0, 1, 2, 3, x, y\}$
- A string over an alphabet: a finite sequence of symbols from the alphabet

$$w = x1211y3$$

- Length of a string w : $|w|$
- Empty string: ϵ

Strings and Languages

- Reverse of w : w^R
- Substring
- Concatenation

$$w = w_1w_2w_3 \quad v = v_1v_2 \\ \longrightarrow wv = w_1w_2w_3v_1v_2$$

- Lexicographic ordering of strings

$$\Sigma = \{0, 1\}, \quad (\epsilon, 0, 1, 00, 01, 10, 11, 000, \dots)$$

- Language : a set of strings