

Pumping Lemma

Properties of Regular Languages

- Limitation of FSM — finite number of states
- All regular languages have a special property — pumping lemma.
- Theorem Pumping lemma

Let L be regular. Then $\exists n$ such that $\forall s \in L$ with $|s| \geq n$, there exists $x, y, z \in \Sigma^*$ such that

1. $s = xyz$
2. $|xy| \leq n$
3. $|y| \geq 1$
4. $\forall i \geq 0, xy^iz \in L$

- Proof:

Prove Non-regularity

- Use pumping lemma
- Examples
 - $L = \{0^n 1^n \mid n \geq 0\}$
 - $L = \{ww \mid w \in \{0, 1\}^*\}$
 - $L = \{0^n \mid n \text{ is prime}\}$

Summary

- Showing regular
 - construct DFA, NFA
 - construct regular expression
 - show L is the union, concatenation, intersection, \dots (regular operations) of regular languages.
- Showing non-regular
 - pumping lemma
 - assume regular, apply closure properties of regular languages and obtain a known non-regular language.