# Nondeterminism

# Nondeterministic Computation

- DFA (deterministic FA):

  the next state is determined exactly based on the current state and the next input symbol.

- NFA (nondeterministic FA):

  several choices may exists for the next state.

- NFA is a generalization of DFA. DFA is a special case of NFA.

# Features of NFA

- A state may have 0, 1, or many moves (arrows) for each symbol.
- Arrows may be labeled with $a \in \Sigma$ or $\epsilon$.
- A kind of parallel computation.

# Advantages of NFA

- Smaller
- Easier to construct
- Easier to understand
- Every NFA can be converted into an equivalent DFA.

# More Examples of NFA

- L : the set of all strings that end in either 101 or 011.

# Formal Definition of a NFA

- Similar to DFA except the transition function.
- A NFA is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where

  $Q$: a finite set of states

  $\Sigma$: a finite input alphabet

  $\delta$: $Q \times \Sigma_\epsilon \longrightarrow 2^Q$, the transition function, $\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$

  $q_0 \in Q$: the initial state

  $F \subseteq Q$: the accept/final states
- Example

# Formal Definition of Computation of a NFA

- A formal definition of " N accepts w":
  if $w = y_1 y_2 \cdots y_m$, where $y_i \in \Sigma \cup \{\epsilon\}$,
  and there exists a sequence of states $r_0, r_1, \cdots, r_m$

  1. $r_0 = q_0$
  2. $r_{i+1} \in \delta(r_i, y_{i+1})$ for $i = 0, 1, \cdots, m-1$
  3. $r_m \in F$

# Equivalence

- Two machines are equivalent if they recognize the same language.
- Theorem: Every NFA has an equivalent DFA.

  Proof: proof by construction.

# Proof

Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA, and $L(N) = A$.
We construct a DFA $M = (Q', \Sigma, \delta', q_0', F')$ s.t. $L(M) = A$.

When N has no $\epsilon$ arrows:

1. $Q' = 2^Q$
2. $\delta'$: For each $R \in Q'$ and each $a \in \Sigma$, let
   $\delta'(R, a) = \cup_{r \in R} \delta(r, a)$
3. $q_0' = \{q_0\}$
4. $F' = \{R \in Q' | R \cap F \neq \emptyset\}$

When N has $\epsilon$ arrows:

- $E(R) = \{q | q$ can be reached from $R$ through $\epsilon$ arrows $\}$.
- $\delta'(R, a) = \cup_{r \in R} E(\delta(r, a))$
- $q_0' = E(\{q_0\})$

# Examples of Converting NFAs to DFAs

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# Regular Languages

- Corollary: A language is regular iff some NFA recognize it.
- Return to the theorem:

  The class of regular languages is closed under concatenation.