Regular Languages

Regular Languages

- Corollary: A language is regular iff some NFA recognize it.
- Re-visit the proof of the theorem:

  The class of regular languages is closed under Union.

- Return to the theorem:

  The class of regular languages is closed under concatenation.

  Proof:

Star Operation

- Theorem:

  The class of regular languages is closed under the star operation.

  Proof: proof by construction.

Regular Expressions

- Regular expressions were designed to denote regular languages.
  They are built from a set of primitives using regular operations.
- Example:

$$(0 \cup 1)00^*$$

- Shorthands
  - 0 : { 0 }
  - ϵ : $\{\epsilon\}$
  - $0 \cup 1$: $\{0\} \cup \{1\}$
  - 01: $0 \circ 1$
  - 0^*: $\{0\}^*$
  - Σ: $(0 \cup 1)$, if $\Sigma = \{0, 1\}$

Definition of Regular Expressions

- A regular expression on some alphabet Σ is defined inductively as:
  - $\emptyset$, ϵ, and $a \in \Sigma$ are regular expressions.
  - If P and Q are regular expressions, then
    $P \cup Q$, $P \circ Q$, and P^*
    are regular expressions.
- Examples

Regular Expressions and Finite Automata

- Theorem:

  A language is regular iff some regular expression describes it.

  - Regular expression $\Longrightarrow$ regular language
  - Regular language $\Longrightarrow$ regular expression